RICEVIMENTO STUDENTI 16/10/2026

$A^{a_n} \to A^{\ell}$ SE $a_n \to \ell$

$a_n \to 0 \Rightarrow A^{a_n} \to A^0 = 1$

$a_n \to \ell \Rightarrow \dfrac{A^{a_n}}{A^{\ell}} \to 1$?

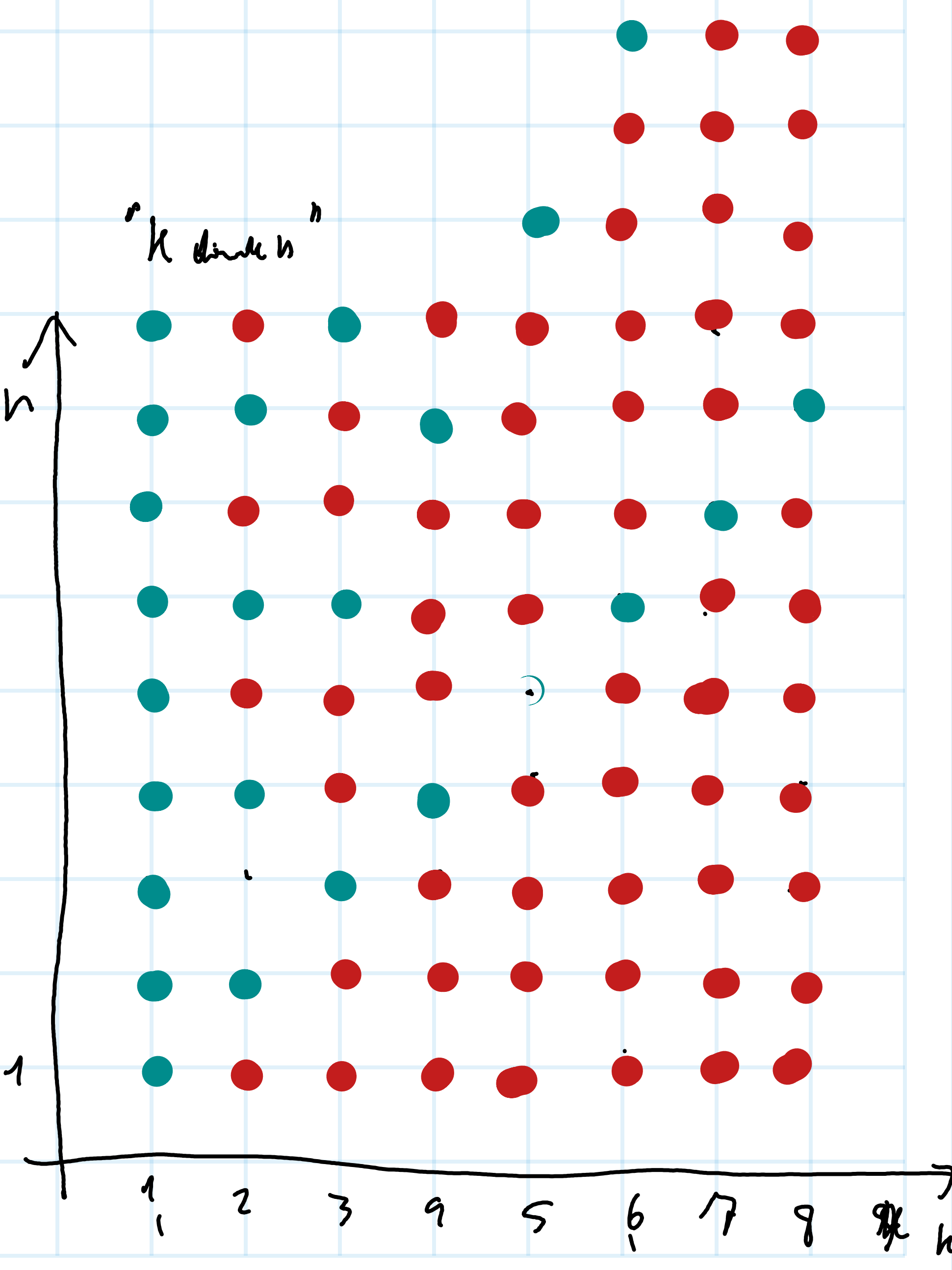
$a_n \to \ell \Rightarrow A^{\overbrace{a_n - \ell}^{c_n}} \to 1$

$c_n = (a_n - \ell)$

$c_n \to 0 \qquad A^{c_n} \to 1$

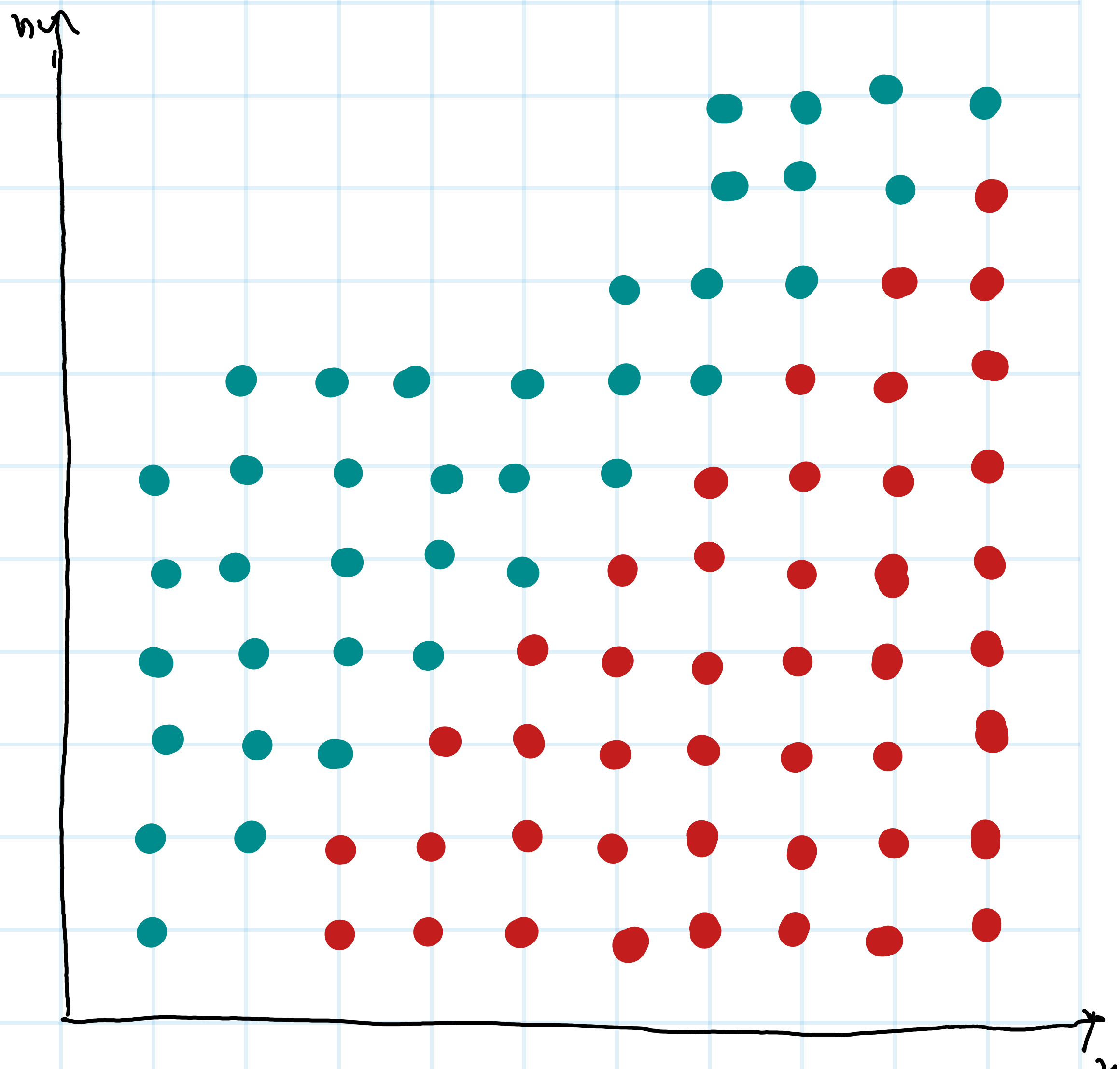
$\forall k \in \mathbb{N}$, FREQ. IN n

k divide n



DEF IN n, DEF IN k $n \geq k$

DEF IN k, DEF IN n $n \geq n$



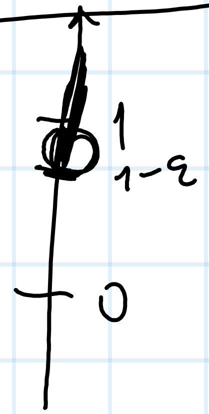
n

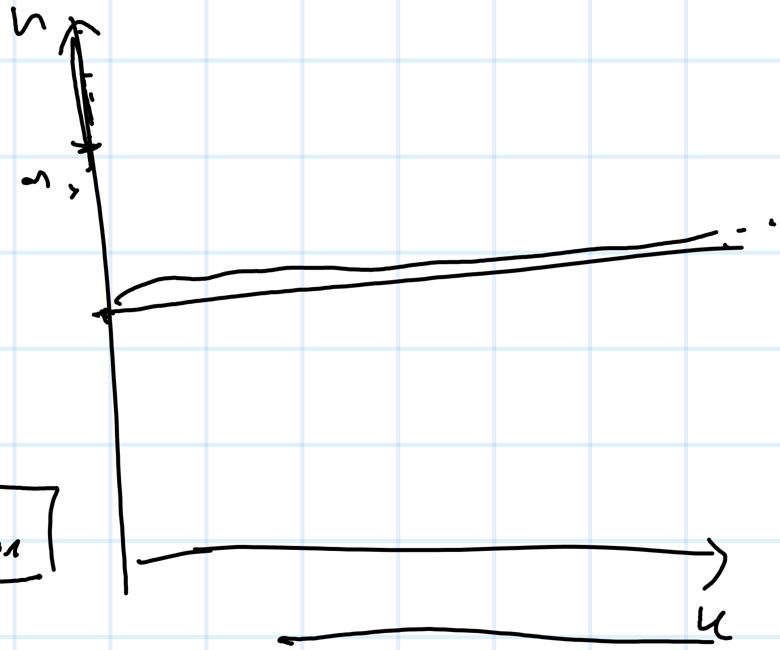
$\frac{n}{n+2023} \to 1$

FREQ IN N DEF IN N

$\forall \varepsilon > 0$ DEF. IN N FREQ IN N $\left|\frac{(-1)^n \cdot (-1)^n\, n}{n+2023}\right| > \cancel{1-\varepsilon} \; \frac{9}{10}$

$\varepsilon = \frac{1}{10}$





$\boxed{\lfloor a_n \rfloor \le a_n < \lfloor a_n \rfloor + 1}$

$0<\alpha<1$

$\left(1+\frac{1}{a_n}\right)^{a_n}$

$(n^n)^\alpha = o(n!)$

$$\lim_{n\to+\infty} \boxed{\frac{n!^{2023}}{(n^n)^{2022}}} = \lim_{n\to+\infty} \left(\underbrace{\frac{n!}{(n^n)^{\frac{2022}{2023}}}}_{\to+\infty}\right)^{2023}$$

$\frac{n!}{(n^n)^\alpha} \to +\infty$

$$\lim_{n\to+\infty} \frac{n! \cdot (n!)^{2022}}{(n^n)^{2022}}$$

$$\lim_{n\to+\infty} \underbrace{\frac{e^n \cdot \cancel{\left(1+\frac{1}{\sqrt{n}}\right)^{\frac{1}{n}}} \cdot (n+1)}{n^{n+1}\cdot\sqrt{n}}}_{a_n} \qquad \frac{(n+1)^{n+1}}{e^{n+1}}\cdot\sqrt{2\pi(n+1)}$$

$$\frac{a_{n+1}}{a_n} = \frac{e^{n+1}\cdot\left(1+\frac{1}{\sqrt{n+1}}\right)^{\frac{1}{n+1}}\cdot(n+2)}{(n+1)^{n+2}\cdot\sqrt{n+1}}\cdot\frac{n^{n+1}\cdot\sqrt{n}}{e^{n}\cdot\left(1+\frac{1}{\sqrt{n}}\right)^{\frac{1}{n}}\cdot(n+1)}$$

$$= \underbrace{\sqrt{\frac{n}{n+1}}\cdot\frac{\left(1+\frac{1}{\sqrt{n+1}}\right)^{\frac{1}{n+1}}}{\left(1+\frac{1}{\sqrt{n}}\right)^{\frac{1}{n}}}\cdot\frac{n+2}{n+1}}_{\downarrow\, 1}\left(\frac{n}{n+1}\right)^{n+1}$$

e

$\downarrow$

e

$$\frac{1}{\left(1+\frac{1}{n}\right)^{n+1}} \to 1$$

$$n! \approx \frac{n^n}{e^n}\cdot\sqrt{2\pi n}$$

5^n

$$\lim_{n\to\infty} \boxed{\frac{(n!)^2\, 4^n}{(2n)!}} = a_n$$

$$\frac{a_{n+1}}{a_n} = \frac{((n+1)!)^2\, 4^{\cancel{n+1}}}{(2n+2)!}\cdot\frac{(2n)!}{(n!)^2\cdot \cancel{4^n}} =$$

$$= \frac{(n+1)^2}{\frac{(2n+2)}{2}\cdot\frac{(2n+1)}{2}} = \frac{(n+1)^{\cancel{2}}}{(\cancel{n+1})(n+\frac{1}{2})}$$

$$\frac{a_{n+1}}{a_n} = \dots = \frac{n+1}{n+\frac{1}{2}} = \frac{2n+1+1}{2n+1} = \boxed{1+\frac{1}{2n+1}} \to 1$$

$$\frac{a_{n+1}}{a_n} > \frac{b_{n+1}}{b_n}$$

$$a_n = n$$

$$\frac{a_{n+1}}{a_n} = \frac{n+1}{n} = 1+\frac{1}{n} \to 1$$

$$b_n = \sqrt{n-1}$$

$$\frac{b_{n+1}}{b_n} = \frac{\sqrt{n+1}}{\sqrt{n}} = \sqrt{1+\frac{1}{n}}$$

$$\sqrt{\frac{n-1+1}{n-1}} =$$

$$= \sqrt{1+\frac{1}{n-1}} \quad 1+\frac{1}{2n+1} \overset{?}{>} \sqrt{1+\frac{1}{n}}$$

$$\left(1+\frac{1}{2n+1}\right)^2 \overset{?}{>} 1+\frac{1}{n}$$

$$\cancel{1}+\frac{2}{2n+1}+\left(\frac{1}{2n+1}\right)^2 \overset{?}{>} \cancel{1}+\frac{1}{n}$$

$$\frac{1}{n}-\frac{2}{2n+1} \overset{?}{<} \left(\frac{1}{2n+1}\right)^2$$

$$\frac{\cancel{2n}+1-\cancel{2n}}{n(2n+1)} \overset{?}{<} \frac{1}{(2n+1)(2n+1)}$$

$$\frac{1}{n\cancel{(2n+1)}} \overset{?}{<} \frac{1}{(2n+1)^{\cancel{2}}}$$

$$\boxed{b_n=\sqrt{n+1}}$$

$$\frac{a_{n+1}}{a_n}=1+\frac{1}{2n+1}$$

$$\frac{b_{n+1}}{b_n}=\sqrt{1+\frac{1}{n+1}}$$

$$1+\frac{1}{2n+1} \overset{?}{>} \sqrt{1+\frac{1}{n+1}}$$

$$\cancel{1} + \frac{2}{2n+1} + \frac{1}{(2n+1)^2} > \cancel{1} + \frac{1}{n+1}$$

$$\frac{1}{n+1} - \frac{2}{2n+1} < \frac{1}{(2n+1)^2}$$

$$\frac{2}{2n+2} - \frac{2}{2n+1}$$

$$\boxed{\frac{b_{n+1}}{b_n} < \frac{a_{n+1}}{a_n} \qquad n \geq n_0}$$

$$b_n \to +\infty$$

$$\Downarrow$$

$$a_n \to +\infty$$

$$\boxed{\frac{b_{n_0+k}}{b_{n_0}}} = \frac{b_{n_0+1}}{b_{n_0}} \cdot \frac{b_{n_0+2}}{b_{n_0+1}} \cdot \frac{\cdot}{b_{n_0+1}} \cdots \frac{b_{n_0+k}}{b_{n_0+k-1}}$$

$$\leq \frac{a_{\cancel{n_0+1}}}{a_n} \cdot \frac{a_{n_0+2}}{a_{\cancel{n_0+1}}} \cdots \frac{a_{n_0+k}}{\cancel{b_{n_0+k-1}}}$$

$$= \frac{a_{n_0+k}}{a_{n_0}}$$

$$\frac{b_{n_0+k}}{b_{n_0}} \leq \frac{a_{n_0+k}}{a_{n_0}}$$

$$\boxed{a_{n_0+k}} \geq \boxed{\frac{a_{n_0}}{b_{n_0}}} b_{n_0+k} \to \infty$$