Analisi Matematica 1 - 23 Settembre 2024 - docente: Callegari

Lezione 1: Prime nozioni sugli insiemi

DEF.1 L'INSIEME A È VUOTO SE NON GLI APPARTIENE ALCUN ELEMENTO.

DEF.2 DATI 2 INSIEMI A, B DIREMO CHE

$A \subset B$ SE ogni elemento di A È ANCHE ELEMENTO DI B.

$A = B$ SE $A \subset B$ E $B \subset A$

$$A = \{1, 3, 7\}.$$

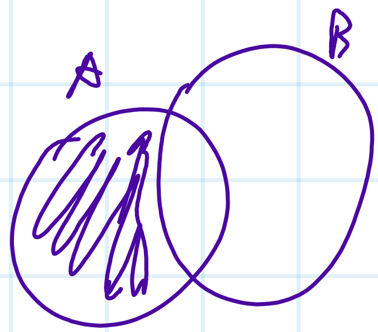
$$B = \{\underline{x \in \mathbb{Z}} \mid \underline{2 \text{ divide } x}\}$$

$$A = \left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots, \frac{1}{n}, \dots \right\}$$

$$A = \left\{ \frac{1}{n} \;\middle|\; n \in \mathbb{N} - \{0\} \right\}$$

$$A = \{ f(n) \mid n \in B \}$$

$\boxed{\text{DEF.}}$ DATI 2 INSIEMI $A, B \subset U$ DEFINIAMO

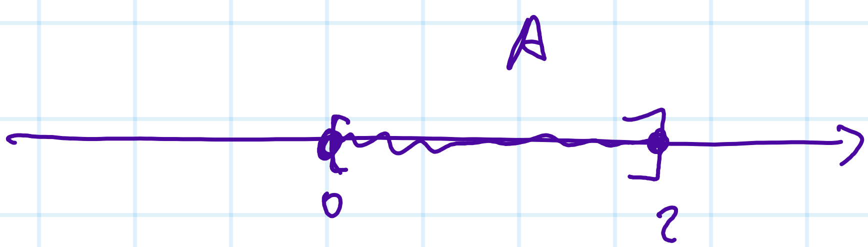


$$A \cup B = \{ x \in U \mid x \in A \text{ o } x \in B \}$$

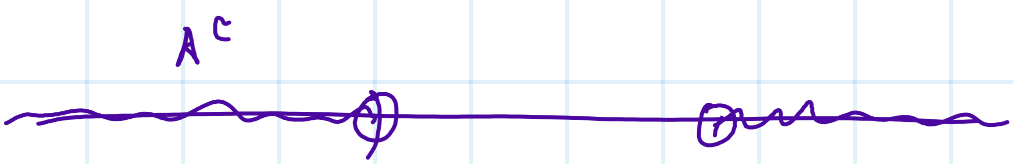
$$A \cap B = \{ x \in U \mid x \in A \text{ e } x \in B \}$$

→ $$A - B = \{ x \in U \mid x \in A \text{ MA } x \notin B \}$$

$$A \times B = \{ (x,y) \mid x \in A \text{ e } y \in B \}$$



$$A^c = (-\infty, 0) \cup (2, +\infty)$$

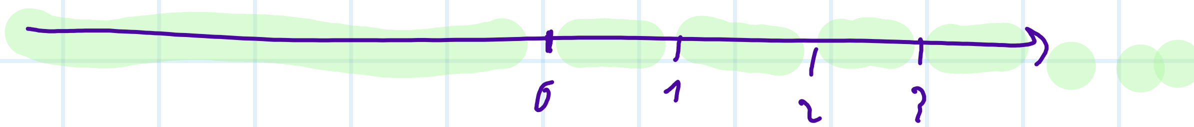


DEF. DATO $A \subset U$ DEFINIAMO $A^c = U - A$

$\mathbb{N}$

$\mathbb{N}^c$

$\mathbb{R}$



$U = \mathbb{R}$ $\qquad \mathbb{N}^c = (-\infty, 0) \cup (0,1) \cup (1,2) \cup \dots \cup (n, n+1) \cup \dots$

$U = \mathbb{Z}$ $\qquad \mathbb{N}^c = \{-1, -2, -3, \dots, -n, \dots\}$

$\boxed{T.1}$ (DISTRIBUTIVITÀ)

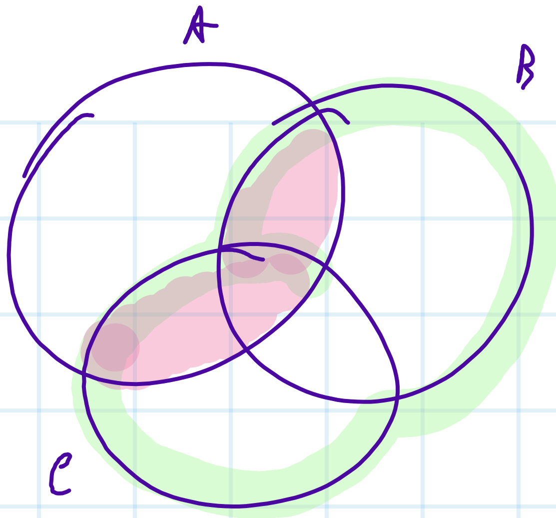
DATI $A, B, C \subset U$

ALLORA

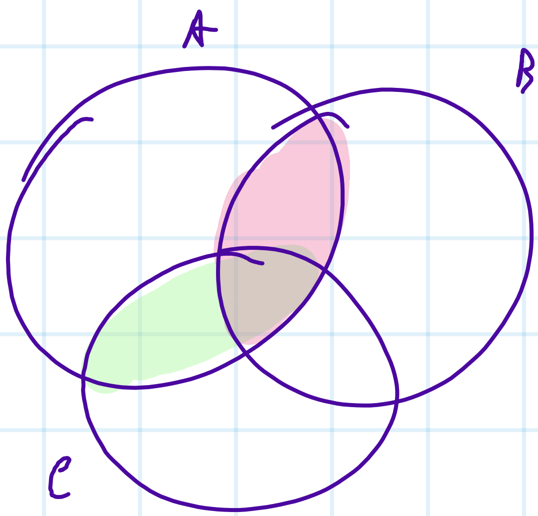
$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

$A \cap (B \cup C)$



$(A \cap B) \cup (A \cap C)$



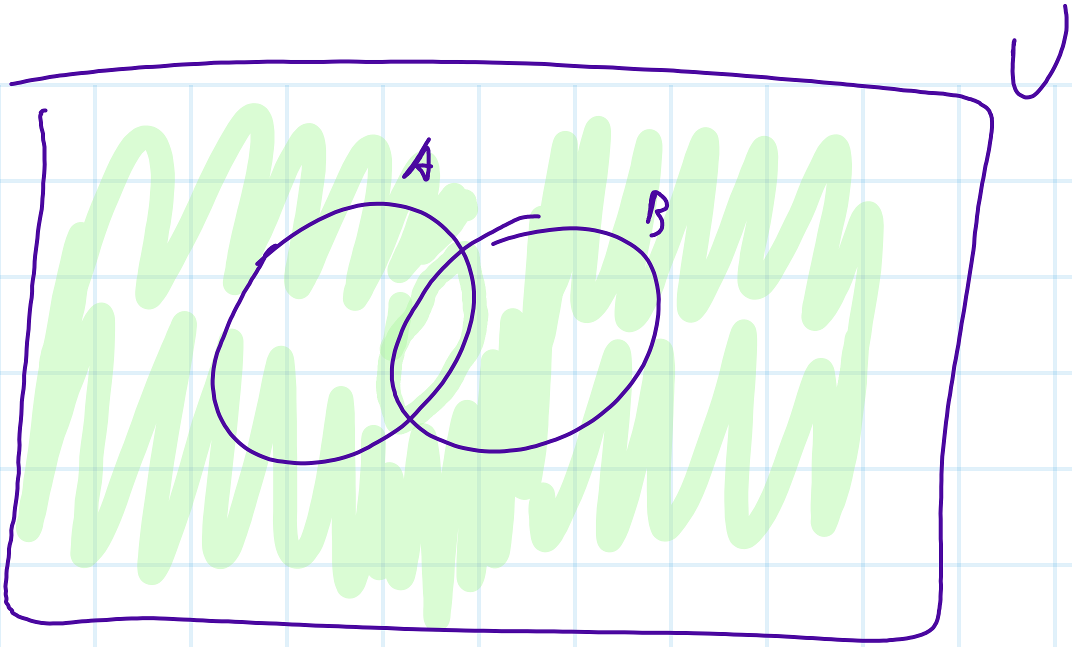
$\boxed{T.2}$ DATI $A, B \subset U$

SI HA:

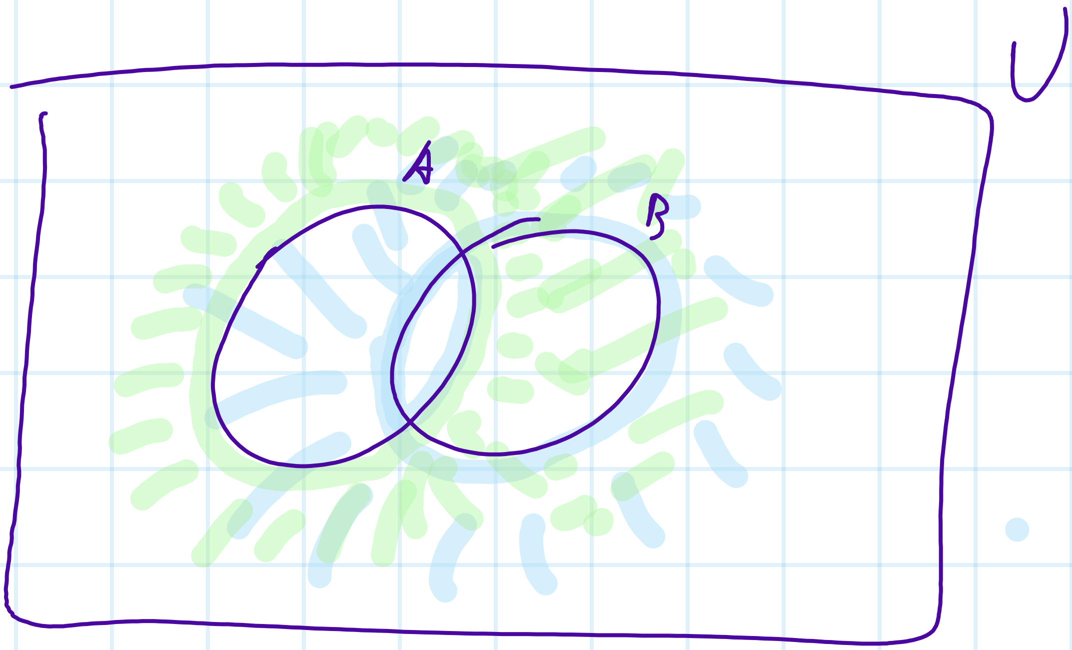
1) $(A \cap B)^c = A^c \cup B^c$

2) $(A \cup B)^c = A^c \cap B^c$

$(A \cap B)^c$



$A^c \cup B^c$



$\mathcal{F} = \{A_i\}_{i \in I} \qquad A_i \subset U \quad \forall i \in I$

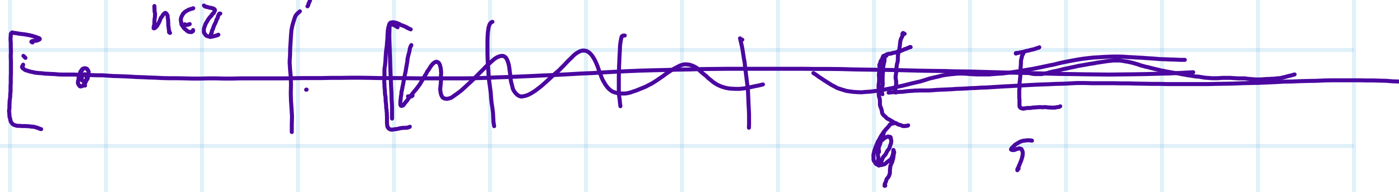
$\bigcup_{i \in I} A_i = \{x \in U \mid \exists i \in I \text{ t.q. } x \in A_i\}$

$\bigcap_{i \in I} A_i = \{x \in U \mid \forall i \in I \ x \in A_i$

$U = \mathbb{R}$

$$A = \bigcup_{n \in \mathbb{Z}} [n, +\infty) = \mathbb{R}$$

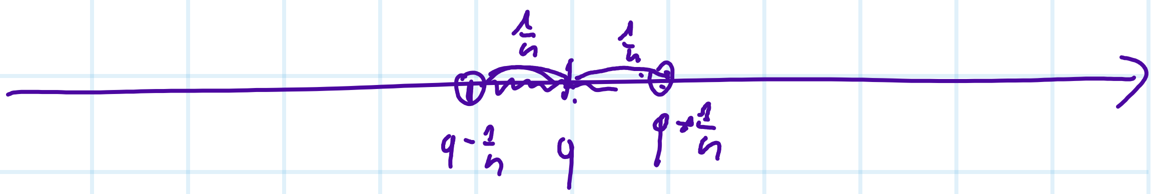
$$B = \bigcap_{n \in \mathbb{Z}} [n, +\infty) =$$



$$\bigcap_{n \in \mathbb{N} - \{0\}} \left(\bigcup_{q \in \mathbb{Q}} \left(q - \frac{1}{n}, q + \frac{1}{n}\right) \right) = ?$$

$$\rightarrow \quad \bigcup_{q \in \mathbb{Q}} \left(\overbrace{\bigcap_{n \in \mathbb{N} - \{0\}} \left(q - \frac{1}{n}, q + \frac{1}{n}\right)}^{A_q} \right) = ?$$

$\left(q - \frac{1}{n}, q + \frac{1}{n}\right)$



$n=1$

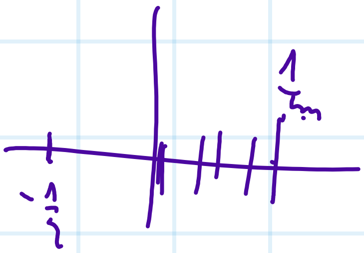
$$\bigcup_{n\in\mathbb{N}-\{0\}}\left(\boxed{\bigcap_{m\in\mathbb{N}-\{0\}}\left(-\frac{1}{n},\frac{1}{m}\right)}\right)=$$

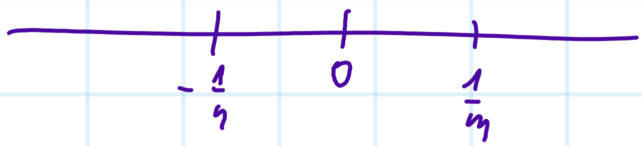
$(-\frac{1}{n},0]$

$$\overbrace{\bigcap_{m\in\mathbb{N}-\{0\}}\left(-\frac{1}{2},\frac{1}{m}\right)}=\left(-\frac{1}{2},0\right]$$

$$\overbrace{\bigcap_{m\in\mathbb{N}-\{0\}}\left(-1,\frac{1}{m}\right)}=(-1,0]$$

$$=\bigcup_{n\in\mathbb{N}-\{0\}}\left(-\frac{1}{n},0\right]=(-1,0]$$





$$\bigcap_{m\in\mathbb{N}-\{0\}}\left(\overbrace{\bigcup_{n\in\mathbb{N}-\{0\}}\left(-\frac{1}{n},\frac{1}{m}\right)}^{(-1,\frac{1}{m})}\right)=$$

$$=\bigcap_{m\in\mathbb{N}-\{0\}}\left(-1,\frac{1}{m}\right)=(-1,0]$$

$$\bigcup_{q\in\mathbb{Q}}\left(\overbrace{\bigcap_{n\in\mathbb{N}-\{0\}}\left(q-\frac{1}{n},q+\frac{1}{n}\right)}\right)=\bigcup_{q\in\mathbb{Q}}\{q\}=\mathbb{Q}$$

$\mathbb{Q}$

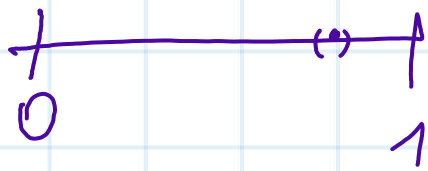
$$\bigcap_{n\in\mathbb{N}\setminus\{0\}} \Big(\overbrace{\bigcup_{q\in\mathbb{Q}} \left(q-\tfrac{1}{n},\, q+\tfrac{1}{n}\right)}^{\mathbb{R}} \Big) = \bigcap_{n\in\mathbb{N}\setminus\{0\}} \mathbb{R} = \mathbb{R}$$

$$\rightarrow \boxed{A = \{\sqrt{n} - \lfloor \sqrt{n} \rfloor \mid n \in \mathbb{N}\}}$$

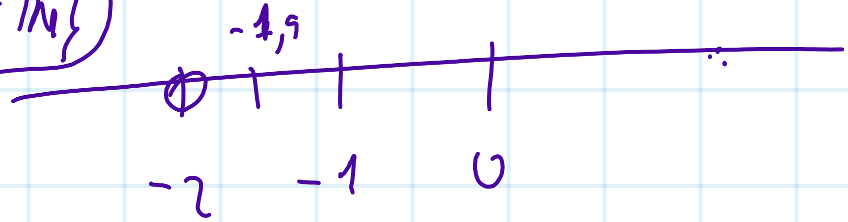
$$\sqrt{n} - \lfloor \sqrt{n} \rfloor$$

$$x - \lfloor x \rfloor$$

$$\lfloor \sqrt{2} \rfloor = 1$$



$$B = \{\sqrt{n} - \sqrt{m} \mid n,m \in \mathbb{N}\}$$



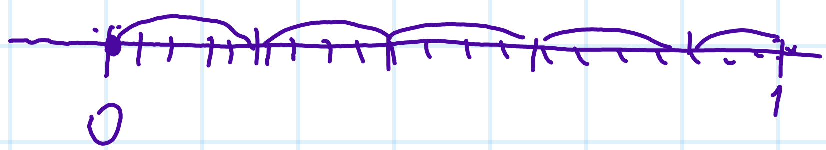
$$A = \{\sqrt{n} - \lfloor \sqrt{n} \rfloor \,/\, n \in \mathbb{N}\}$$

$\sqrt{4} = 2$

$\sqrt{5} = 2,\ldots$

$\ldots\ldots$ $\sqrt{8} = 2,\ldots$ $\sqrt{9} = 3$

$\sqrt{5}, \sqrt{6}, \sqrt{7}, \sqrt{8}, \sqrt{9}$



$100 \, . \, . \, . \, . \qquad 121$

$k = 10$

k^2

k^2+1

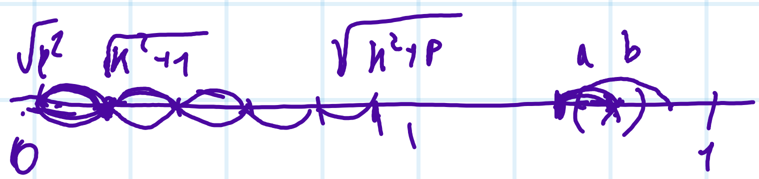
$\vdots$

k^2+2k

$k^2+2k+1 = (k+1)^2$

$0 < a < b < 1$

$\exists n \in \mathbb{N}$ t.c. $\sqrt{n} - \lfloor \sqrt{n} \rfloor \in (a,b)$



$\delta = b - a > 0$

$1 \le p \le 2k$

$$\frac{\sqrt{k^2+1} - \sqrt{k^2}}{1} = \frac{k^2+1-k^2}{\sqrt{k^2+1}+\sqrt{k^2}} = \frac{1}{\sqrt{k^2+1}+\sqrt{k^2}} =$$

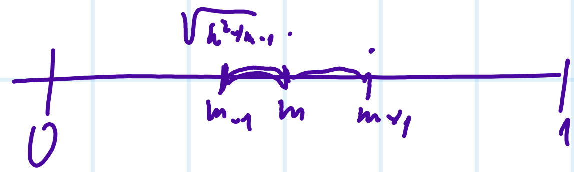
$$= \frac{1}{\sqrt{k^2+1}+k} < \frac{1}{2k}$$

$\exists k \in \mathbb{N}$. t.c. $\sqrt{k^2+1} - k < \min\{\delta, a\}$

Sia $p \in \{1, 2, \ldots, 2n\}$ t.c.

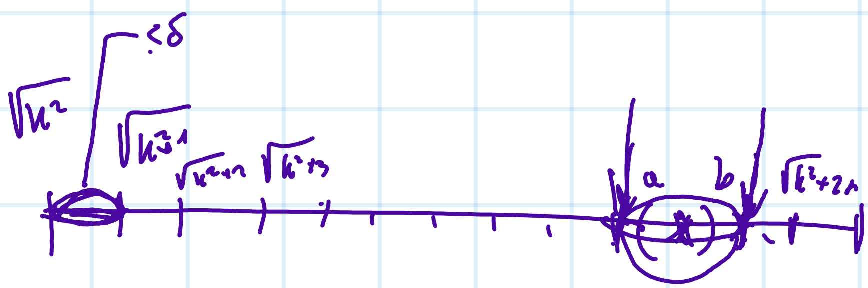
$$p = \max\{ i \in \{1, \ldots, 2n\} \,/\, \sqrt{n^2+i} < a \}$$

$p > 1$



$$\sqrt{n^2+m} - \sqrt{n^2+m-1} \overset{?}{\geq} \sqrt{n^2+m+1} - \sqrt{n^2+m}$$

$$\frac{1}{\sqrt{n^2+m} + \sqrt{n^2+m-1}} \qquad \frac{1}{\sqrt{n^2+m+1} + \sqrt{n^2+m}}$$



$\sqrt{n^2+1} - \sqrt{n^2}$

$\sqrt{n^2+p}$

$\sqrt{n^2+p+1}$

$$\sqrt{n^2+p+1} - \sqrt{n^2+p} > b - a > \delta$$