Università di Roma Tor Vergata - Facoltà di ingegneria

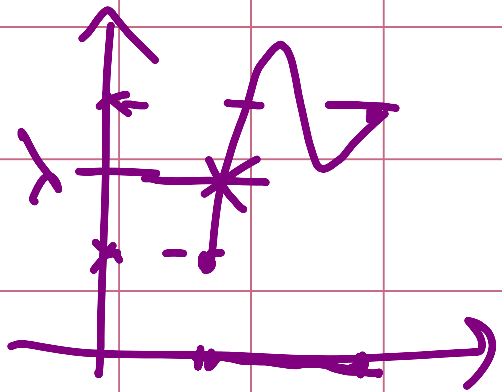
Corso di Analisi Matematica II per ingegneria Gestionale

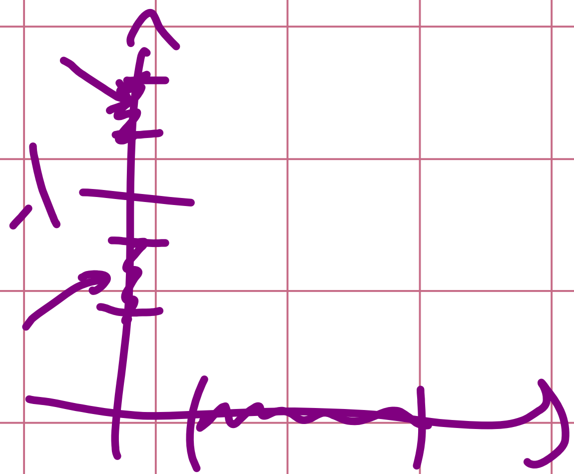
Docente prima parte: **Emanuele Callegari** (Univ. di Roma *Tor Vergata*) - Sito di riferimento per il materiale: www.problemisvolti.it

17 Ottobre 2013

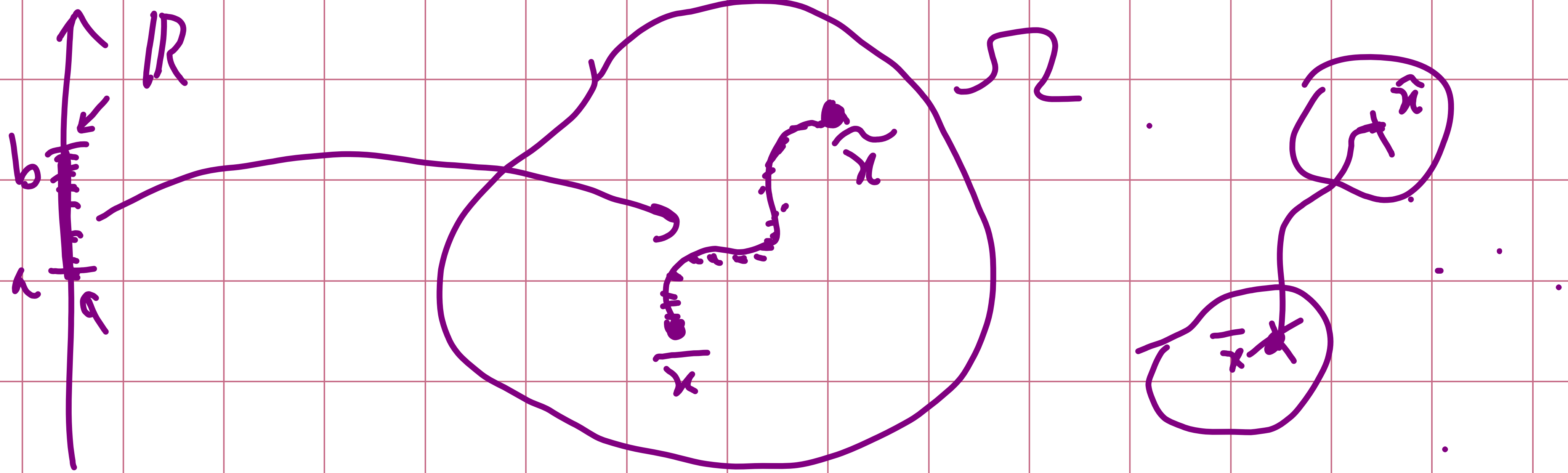
Lezione 9: LIMITI IN $\mathbb{R}^n$ (IV Parte)

Teorema (valori intermedi in $\mathbb{R}$, invece in $\mathbb{R}^n$ ... ...)





$\boxed{\text{Def.}}$ Dato $\Omega \subset \mathbb{R}^n$ diremo che Ω è connesso per archi se, $\forall\, \bar{x}, \hat{x} \in \Omega$ $\exists\, \gamma : [a,b] \to \Omega$ continua t.c. $\gamma(a) = \bar{x}$ e $\gamma(b) = \hat{x}$.

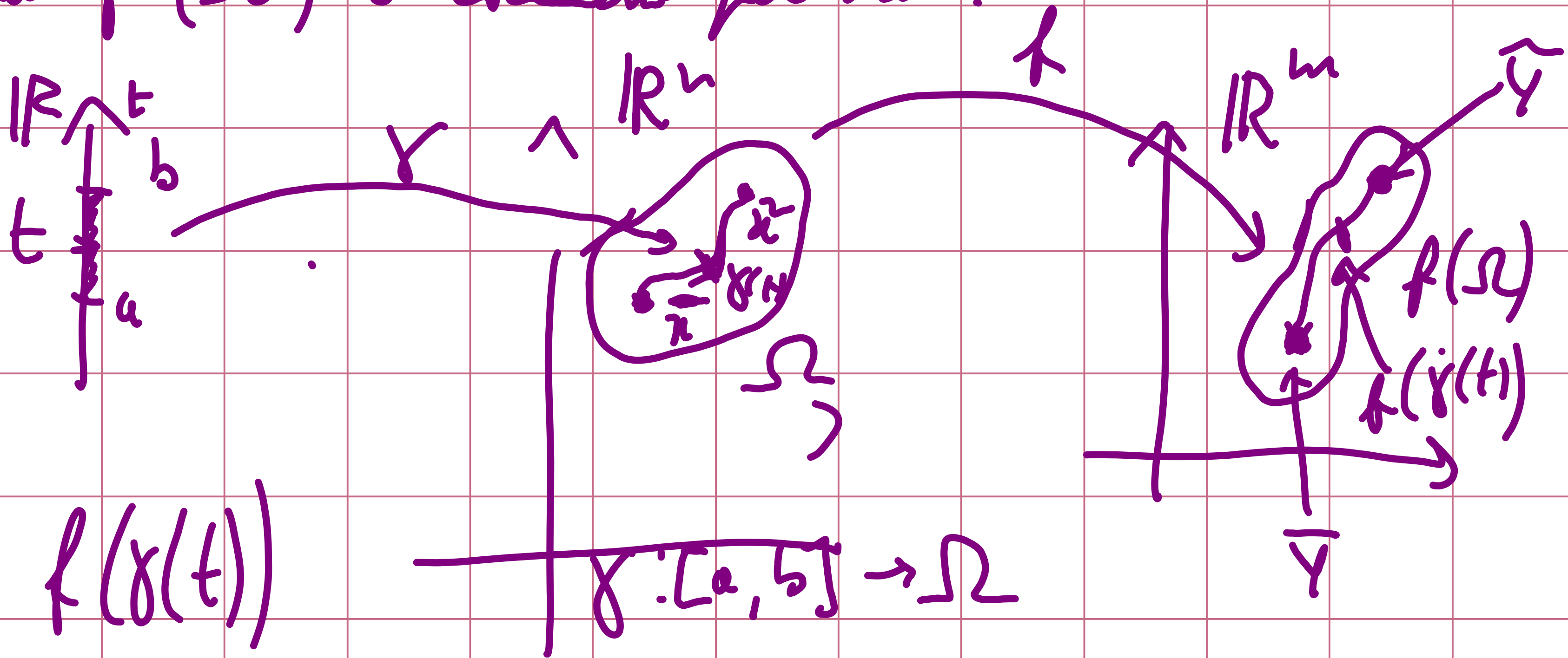


$\boxed{\text{Teorema}}$

Dati $\Omega \subset \mathbb{R}^n$, $f : \Omega \to \mathbb{R}^m$ continua, e sia Ω connesso

Allora anche $f(\Omega)$ è connesso per archi.

Dim

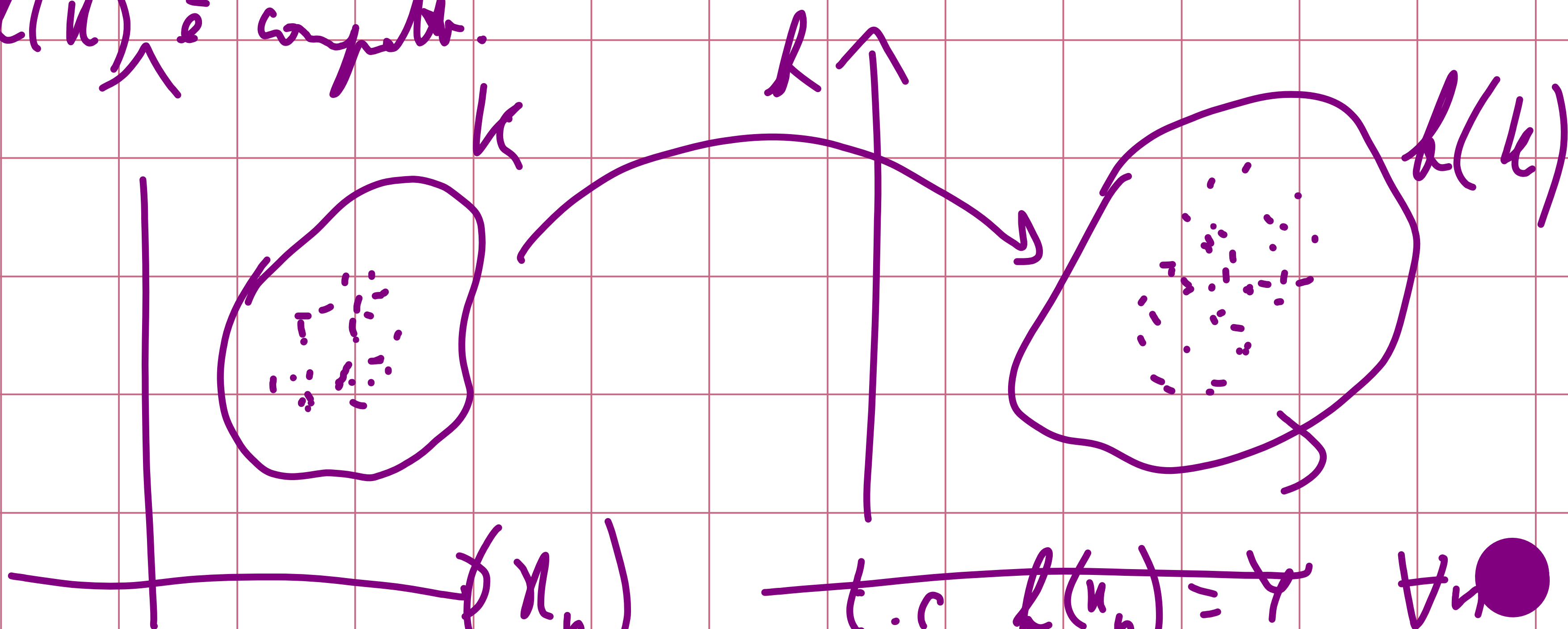


$\gamma(a) = \bar{x} \quad \gamma(b) = \tilde{x}$

Teorema (Weierstrass)

Sia $K \subset \mathbb{R}^n$ compatto e $f : K \to \mathbb{R}^m$ continua.

Allora $f(K)$ è compatto.



$(y_n)_{n\in\mathbb{N}} \subset f(K)$ $\Rightarrow$ $(x_n)_{n\in\mathbb{N}}$ t.c. $f(x_n) = y_n$ $\forall n$

Prendo $(x_{n_k})_{k\in\mathbb{N}}$ sottosucc. di (x_n) t.c. $x_{n_k} \to \bar{x} \in K$

Prendo la corrispondente (y_{n_k}) e quindi, $\forall k \in \mathbb{N}$

$f(x_{n_k}) = y_{n_k}$ ma allora $y_{n_k} \to f(\bar{x})$

perché f è continua e applico il T. Ponte

$\boxed{\text{Esempio 1}}$

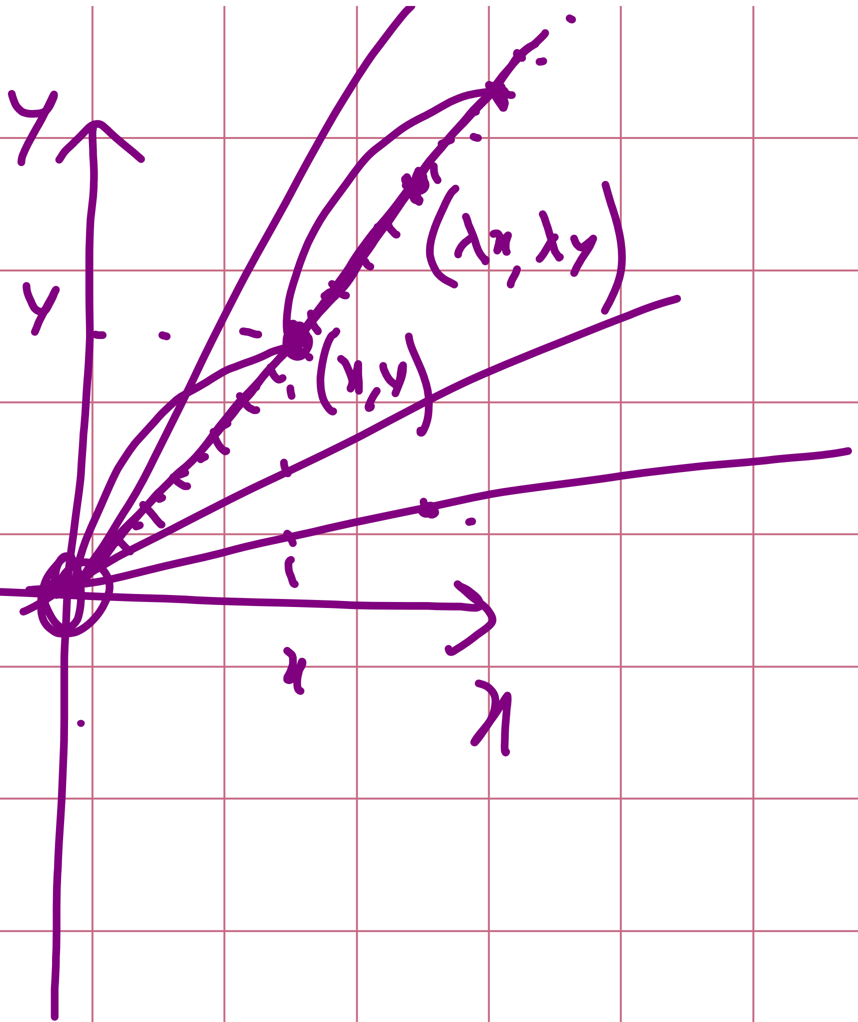
$$\lim_{(x,y)\to(0,0)} \frac{x^7y^2}{x^8+y^8} = 0$$

$$\frac{x^7y^2}{x^8+y^8} = \underbrace{\boxed{\frac{x^7 y}{x^8+y^8}}}_{\text{limitata}} \, \underbrace{y}_{\to 0}$$

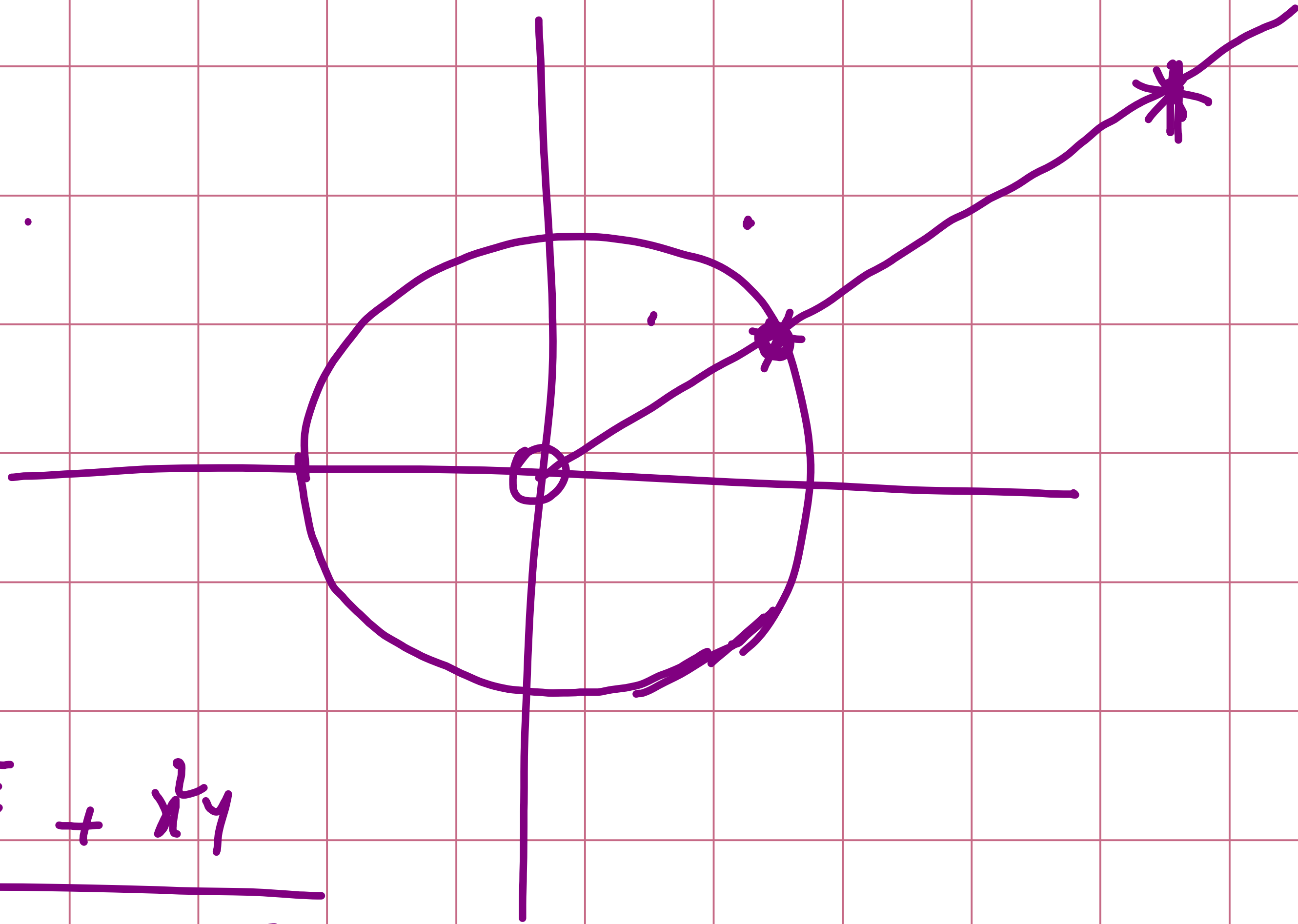
$$\boxed{g(x,y) = \frac{x^7 y}{x^8+y^8}}$$

$$g(\lambda x, \lambda y) = \frac{(\lambda x)^7 \cdot \lambda y}{(\lambda x)^8 + (\lambda y)^8} =$$

$$= \frac{\cancel{\lambda^8}\, x^7 \cdot y}{\cancel{\lambda^8}\,(x^8+y^8)} = g(x,y)$$



$$g(x,y) = \frac{x^7 y}{x^8 + y^8}$$



$$h(x,y) = \frac{x\sqrt{x^4+y^4} + x^2 y}{|x^3| + \sqrt{y^6}}$$

$$h(\lambda x, \lambda y) = \frac{\lambda x\sqrt{(\lambda x)^4 + (\lambda y)^4} + (\lambda x)^2 \cdot \lambda y}{|(\lambda x)^3| + \sqrt{(\lambda y)^6}} = \boxed{\lambda > 0}$$

$$= \frac{\cancel{\lambda^3}\left(x\sqrt{x^4+y^4} + x^2 y\right)}{\cancel{\lambda^3}\left(|x^3| + \sqrt{y^6}\right)} = h(x,y)$$

Esempio 2

$$\lim_{(x,y)\to(0,0)} \frac{e^{x^2y}-1}{x^2+y^2} =$$

$$= \lim_{(x,y)\to(0,0)} \left[\frac{e^{x^2y}-1}{x^2y} \cdot \frac{x^2}{x^2+y^2} \cdot y \right] = 0$$

$$\quad 1 \qquad \text{limitata} \qquad 0$$

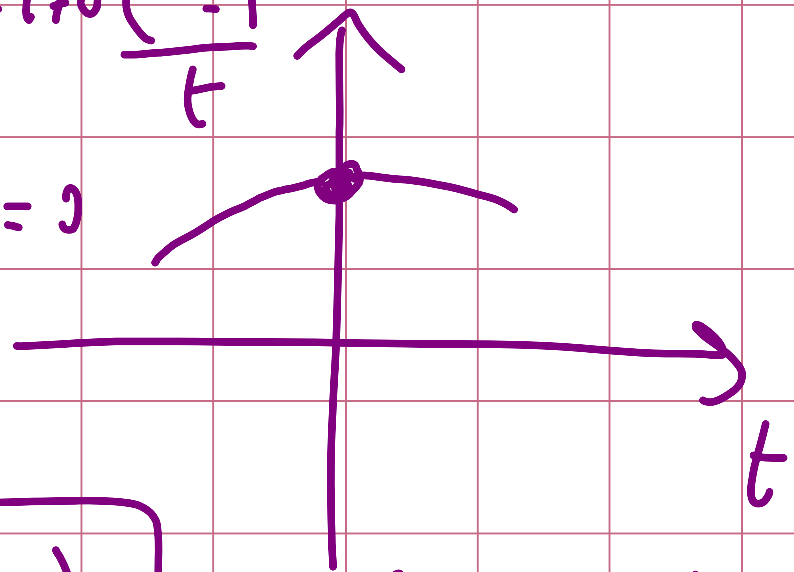
N!

??

$$\lim_{t\to 0} \frac{e^t-1}{t} = 1$$

$$h(t) = \begin{cases} \dfrac{e^t - 1}{t} & \text{per } t \neq 0 \\ 1 & \text{per } t = 0 \end{cases}$$

$\dfrac{e^t - 1}{t}$



$$\boxed{\lim_{(x,y)\to(0,0)} h(x^2 y)} = \lim_{t \to 0} h(t) = 1$$

Esempio 3

(proprietà degli altri)

$$\lim_{(x,y)\to(0,0)} \frac{e^{x^6y^2}-1}{x^8+y^6} =$$

$$= \lim_{(x,y)\to(0,0)} \frac{e^{x^6y^2}-1}{x^6y^2} \cdot \frac{x^6|y|^{\frac{3}{2}}}{x^8+y^6} \cdot \sqrt{|y|} = 0$$

limite

$$= \text{limite} \sqrt{\frac{(x^4)^{\frac{3}{2}} \cdot (|y|^3)^{\frac{1}{2}}}{(x^4)^2 + (|y|^3)^2}} \, (|y|^3)^{\frac{1}{6}}$$

$$\frac{1}{2} + \frac{1}{6} = \frac{3+1}{6}$$

$$\boxed{\frac{a^{\frac{3}{2}}\, b^{\frac{1}{2}}}{a^2 + b^2}}$$